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INTRODUCTION

Experiments consisting of the analysis of light backscattered from different heights in the atmosphere, have been carried out many times since E. O. Hulbert successfully used this technique to measure atmospheric density from the ground in 1937. In later years, however, new possibilities were opened by the advent of lasers. The experiment was repeated and new problems came to light.

G. Fiocco and L. D. Smullin made and published a series of observations, at M.I.T., in 1963. Their results based on data gathered in four consecutive nights, show evidence of a highly reflecting layer at about or slightly above a height of 100 km.

Since the implication was that some particulate matter was present in the atmosphere at this altitude, other scientists set out to detect it from other locations. Prof. R. W. H. Wright succeeded in building a somewhat more sensitive equipment at the University of the West Indies, and is currently carrying out and refining the experiment. So far, however, he found no evidence of a reflecting layer at the indicated altitude.

It is therefore of interest to extend these observations to other regions and perform them operationally for a period of at least one year, in order to determine if there is a local or a seasonal character associated with Fiocco's findings. This is the first aim of the present project and, in keeping with the foregoing comments, the equipment will be made as suitable as possible for prolonged routine operation, and, if possible, for easy transportation to different locations.

It must be pointed out, however, that an optical radar sensitive enough to probe the hypothetical layer at 100 km, is also adequate for many observations at lower levels, such as density measurements from molecular Rayleigh scattering, nocturnal detection of thin cirrus clouds, atmospheric opacity and perhaps pollution, refractive index variations, etc. In fact, some of these problems are also being studied in this Institute. They will not, however, be considered in the present report.

October 29, 1965

Light Backscattering Experiment

Given a light source transmitting a light pulse of energy content E_o joules and duration τ seconds, aimed vertically at the sky, the power flux through a layer at height z meters will be:

(1) $$F = \frac{E_o}{\tau} \times \frac{T(z,\lambda)}{A(z)} \qquad \frac{\text{watts}}{m^2} \qquad (A(z) \text{ in } m^2)$$

where $T(z,\lambda)$ is the attenuation integrated over the one-way path and

(1a) $A(z) = \Omega z^2$ is the area of the layer illuminated by the beam - i.e., the cross section of the beam - when the latter has a solid angle Ω steradians

(1b) The energy of one photon is $E_{ph} = \frac{hc}{\lambda}$ where λ is the wavelength in meters, $c = 3 \times 10^8$ m/s and h is the Planck constant: $h = 6.6 \times 10^{-34}$ joule-sec.; thus E_{ph} is in joules. We can now write eq. (1) as the photon flux through the layer:

(2) $$F_{ph} = \frac{1}{hc} \times \frac{E_o \lambda}{\tau} \times \frac{T(z,\lambda)}{A(z)} \qquad \frac{\text{photons}}{m^2 s}$$

If the thickness of the layer is larger than the pulse length τc, it will behave as a volume target, the amplitude of the signal received at any instant being determined by the summation of the backscattering of all the particles contained in one-half of the pulse volume. Thus, if the layer contains n particles per m^3, all of the same backscattering cross-section σ, then the total cross-section of the layer will be given by:

(3) $$\sigma_t = \frac{V}{2} n\sigma = \frac{\tau c}{2} A(z) n\sigma$$

where σ_t and σ are in m^2. The total rate of photons backscattered by the layer is then:

(4) $$F_s = F_{ph}\sigma_t = \frac{1}{2h} E_o\lambda\, T(z,\lambda)\, n\sigma \qquad \text{photons/s}$$

By definition of backscattering cross-section, the scattering object is treated as a fictitious isotropic source (see D. Kerr, Propagation of Short Radio Waves, Radiation Lab. series, vol. 13, p. 33). Therefore, the backscattered photon flux received on the ground is:

(5) $$F_r = \frac{F_s}{4\pi z^2} = \frac{1}{8\pi h} E_o\lambda \frac{T^2(z,\lambda)}{z^2} n\sigma \qquad \frac{\text{photons}}{\text{m}^2\text{s}}$$

and the rate at which photons are effectively available at the receiver input is:

(6) $$N_o = \frac{1}{8\pi h} \lambda E_o A_r \frac{T^2}{z^2} n\sigma \qquad \text{photons/s}$$

where A_r is the collecting area of the receiving mirror in m^2. Since there will be some loss in the receiver optics, and since the detector will operate with some quantum efficiency smaller than unity, a factor $\alpha < 1$ must be added; the effective photon counting rate then becomes

(7) $$N = 6 \times 10^{31} \lambda E_o A_r \alpha \frac{T^2(z,\lambda)}{z^2} n\sigma \qquad \text{counts/s}$$

We must now introduce various assumptions into eq. (7) in order to estimate the expected photon count. The first four factors after the constant are parameters depending on the equipment only. The ratio that follows depends mainly on the path and the last two factors, only on the target. We will begin with the latter, over which we have no control.

Let us regard the reflecting layer as a population of spherical particles of radius $a = 1\mu$, whose concentration is 10 per cubic meter. For a visible wavelength, they are well out of the Rayleigh region. Although it is very inaccurate to expect geometric scattering from them, this may be accepted as a fairly safe simplification since dielectric particles are likely to have larger backscattering cross-sections in the Mie region rather than in the geometric region. Whence we write:

(8) $$n\sigma = n\pi a^2 \approx 3 \times 10^{-11} \frac{m^2}{m^3}$$

The two way attenuation depends essentially on the wavelength and, to a lesser degree (for the range of heights considered here) on the height. The visible spectrum is essentially free from absorption lines for wavelengths shorter than 6770 Å, but the transmission of the atmosphere decreases towards the higher frequencies as a result of Rayleigh molecular scattering. Thus for 0° zenith angle, $T \approx 0.85$ at $\lambda = 0.67\mu$, but $T \approx 0.62$ for $\lambda = 0.4\mu$. This value will therefore be influenced by the choice of our light source.

As for the height of the reflecting layer, it has been variously reported between 90 and 120 km. Let us then take it as 100 km.

If we now substitute the assumed height and cross-section in eq. (7), we have

(9) $$\boxed{N = 1.8 \times 10^{11} \lambda E_0 A_r \alpha T^2(\lambda) \qquad \text{counts/s}}$$

where the transmission of the atmosphere has become, for a fixed height, a function of λ only. Equation (9) is one of the basic design equations, since it gives the signal-power return (for a specific target) as a function of parameters of the equipment.

The next basic consideration is noise, since it will determine the ability of the system to detect weak signals, or the time that will be necessary to achieve detection by statistical procedures.

We tend to call noise any unwanted return that tends to mask our signal. However, such a return, if stable, would constitute no problem, since the signal would be added to it and could be recovered by subtraction. We are therefore only concerned with the time-fluctuation of whatever returns are measured, since it is this fluctuation that will limit the significance of any deviation from an average count.

The output of the receiver will consist of three main components: (1) the signal photons produced by backscatter in the dust layer; (2) the background photons originating in the many sources of the night sky and (3) the dark current of the detector. If we regard all three as photon fluxes obeying the Poisson distribution, they will have a variance equal to their time-average. The total fluctuation - or standard deviation of the total count - will then be the square root of the sum of all three counts.

The photon count at the receiver output will necessarily be integrated over the resolution element of the system, whose minimum length is determined by the pulse duration τ. Thus if the signal at a given range increment is τN counts the total fluctuation will be: $\sqrt{\tau (N + N_b + N_d)}$, where N_b and N_d denote the background and dark-current counts per second. The ratio P of the signal to the fluctuation is a measure of the statistical significance of the measurement. If the return received from one transmitted pulse does not yield an acceptable significance; it can be improved by repeating the experiment u times and adding the results, for then:

$$P = \frac{u\tau N}{\sqrt{u\tau (N + N_b + N_d)}} \tag{10}$$

where P is seen to increase with u. If it is now desired to reach a certain pre-established significance, eq. (10) must be solved for u, with a given value for P. We have:

$$\boxed{u = P^2 \frac{N + N_b + N_d}{\tau N^2}} \tag{11}$$

which gives the number of pulses necessary to achieve a given statistical validity of the measurement. We shall use u as a figure of merit to compare different systems.

In equation (11) we must know N_b and N_d. N_b originates in the brightness of the night sky and is differently reported by various authors. It is frequency-dependent and consists essentially of a continuous curve with emission bands superimposed along most of the spectrum. Also, it is received from all directions. Therefore, if Φ_b is the power received per unit area of the collector, per unit solid angle and per unit frequency bandwidth, at a given wavelength, the background photon flux will be:

$$N_b = \frac{\Phi_b A_r \Omega_r B}{E_{ph}} = \frac{\lambda}{hc} \Phi_b A_r \Omega_r B \quad \frac{\text{photons}}{\text{sec}} \tag{12}$$

where Ω_r is the solid angle viewed by the receiver and B its bandwidth. In the following we shall assume that the beamwidth of the transmitter and the receiver are made equal. Evaluating the constants yields:

(13)

$$\boxed{N_b = 5 \times 10^{24} F_b \lambda A_r \Omega B \qquad \text{photons/s}}$$

where λ is in meters, Ω in steradians and B must be in the same units considered in the value of Φ_b.

For Φ_b, we take the plot in Geophysics Corporation of America's Report 794-5-01, p. 81, with a safety factor of 3, which yields, at λ = 6950 Å:

(14)
$$\Phi_b = 6 \times 10^{-10} \frac{\text{watts}}{\text{m}^2\text{ster Å}}$$

As for N_d, the detector-noise, it must be taken from the manufacturer's literature, but we can accept as a typical average for tubes of different makes that, with its cathode cooled to temperatures of -50°C to -100°C, a good photo-multiplier produces a dark current equivalent to a count of about 100 photons/second.

System Design

The laser itself has been taken as one starting point in the design of our proposed system. Apparently, among commercial devices and within our budgetary limitations, ruby lasers would deliver the greatest energy per pulse and this energy could be about 5 joules. The wavelength would be between 6900 and 7000 Å, therefore we take it as 0.7μ in our formulae. Moreover, we can take $T^2(\lambda) = 0.7$ and Eq. (9) becomes:

(15)
$$N = 4.4 \times 10^5 A_r \alpha \qquad \text{ct/s}$$

On the other hand, the background signal is given by Eq. (13) where we can substitute Eq. (14) and the values given above, whereby we get:

(16)
$$N_b = 2.1 \times 10^9 A_r \Omega B \alpha \qquad \text{ct/s}$$

where we have added an efficiency factor which should be smaller than for N, being now averaged over the pass-band of the filter. However, to be on the safe side, we shall take the same value. Let us further assume that if the optical components of the system are of reasonably good quality -- which, incidentally, excludes most searchlight mirrors -- the losses incurred by them will be negligible compared to those in the filter and phototube. In the filter we can apparently count on a transmission not smaller than 50% down to very narrow bands. In the phototube, even for the best types, the peak detective efficiency does not greatly exceed 4% at the ruby laser frequency. Thus, multiplying this by the filter transmission, we get: $\alpha = 2 \times 10^{-2}$. Our equations (15) and (16) then become:

$$N = 8.8 \times 10^{3} A_r \text{ ct/s} \tag{17}$$

$$N_b = 4.2 \times 10^{7} A_r \Omega B \text{ ct/s} \tag{18}$$

It is now evident that in order to maximize the signal to background ratio, Ω and B must be made as small as practically possible. Narrowing down the filter bandwidth is desirable up to a certain point, but beyond that it becomes a mixed blessing both on account of lower peak-transmission and because the laser frequency is likely to drift with temperature and may therefore get out of the filter range. It is assumed that a bandwidth of 20 Å represents a fair compromise.

Thus eq. (18) becomes:

$$N_b = 8.4 \times 10^{8} A_r \Omega \text{ ct/s} \tag{19}$$

We may now substitute eqs. (17) and (19) into eq. (11).

For this purpose we shall write:

$$N = k_1 A_r \; ; \; N_b = k_2 A_r \Omega \; ; \; N_d = k_3$$

whereupon eq. (11) becomes:

$$u = P^2 \frac{(k_1 + k_2 \Omega) A_r + k_3}{\tau k_1^2 A_r^2} \qquad (20)$$

As long as the equivalent dark-noise input is negligible compared to the other two components, eq. (20) reduces to:

$$u = P^2 \frac{k_1 + k_2 \Omega}{\tau k_1^2 A_r} \qquad (21)$$

It may seem that the most favorable situation obtains when the background count is also made negligible by keeping Ω to a very small value. In that case, we have simply:

$$u = \frac{P^2}{\tau k_1 A_r} \qquad (22)$$

i.e., the validity of the count is only limited by the signal's own fluctuation. But by comparing eqs. (17) and (19) it is found that it takes a field of view as small as about 10^{-7} steradians to fully justify eq. (22), and this value can only be reached with mirrors of astronomical quality, hence very expensive.

If we compare the number u_1 of pulses required by such a mirror, of area A_1, as given by eq. (22), with the number u_2 given by eq. (21) for a cheaper mirror of size $A_2 > A_1$ used with a field of view Ω, we find:

$$\frac{u_2}{u_1} = \frac{(k_1 + k_2 \Omega) A_1}{k_1 A_2}$$

which shows that we shall have:

$$u_2 < u_1 \text{ whenever } \frac{A_2}{A_1} > 1 + \frac{k_2}{k_1}\Omega \qquad (23)$$

In other words, accepting a large background signal may result in an improvement if the size is increased enough to reduce the overall fluctuation as compared to the total count.

This increase in size may actually constitute an economy, for two reasons: first, because a relatively inaccurate mirror may be used, and secondly, because it may make unnecessary the use of an optical system in the transmitter to reduce the laser beam which, it must be remembered, is assumed to have the same width as the receiver field of view.

In our case, for example, an astronomical mirror is available at our Observatory. Its area is $A_r = 0.164 m^2$ (diameter: 18 in.) Thus $N = 1.44 \times 10^3$ ct/s. It is therefore satisfactory to operate it with a field of view $\Omega = 7.85 \times 10^{-7}$ sr (beamwidth: 3.44') to reduce the background to the magnitude of the equivalent dark-current input: $N_b = 108$ ct/s, and we assume $N_d \approx 100$ ct/s. Eq. (11) then yields, for $P = 10$ and $\tau = 10^{-5}$s: (1) $u = 7900$ pulses. The resolution element τ is taken larger than the expected pulse duration, in order to provide some range-integration.

Instead, we propose a mirror of such accuracy that we can operate it with a field of view of 6×10^{-6} sr (beamwidth: 10'). We assume that one or two minutes of arc is sufficient accuracy for that purpose. Then, if $A_r = 0.81 m^2$ (diameter: 40 in.), we find $N = 7.13 \times 10^3$ ct/s and $N_b = 4.1 \times 10^3$ ct/s. And yet, from eq. (11) : $u = 2240$ for the same P and τ as before. Its focal length must be at least 5.75m if the filter located at the focus is to receive no light at an angle greater than 5° from the axis. Then the aperture at the focus, for the specified field of view, will have to be 1.67 cm in diameter, which is compatible with most photomultipliers considered as possible detectors.

(1) Although the pulse-length will probably be shorter, the received signal will be integrated over 10 μs intervals, since this provides a range-resolution of 1500 m, which is sufficient for this experiment.

We propose to use both mirrors in consecutive stages of our experiment. In other words, during the first period we plan to acquire the laser and receiver, and carry out observations with the existing telescope. A simple data presentation system will be put together in our laboratory, based on oscilloscope techniques. The transmitter, receiver and indicator will consist as much as possible of commercially available instruments.

A new collector and a more elaborate data recorder will be the main additions ar a later stage. Design efforts will be oriented towards a semi-mobile system that may conceivably by operated on mountain sites.

Specifications are out for a laser system. In the main, they call for a Q-switched rubylaser capable of giant pulses of energy $E_0 \geq 5$ Joules. The pulse-length should not exceed 50 nanoseconds. The repetition rate must be at least 1/3 pps. The beam divergence should not exceed 3 milliradians and optics should be provided to reduce it to 1 milliradian. Q-switching and shutter arrangements are to be discussed.

The telescope is already in operation at the observatory of West Palm Beach. It is of the Newtonian type and has a mirror 18 in. in diameter, with a focal length of about 4 m. The beamwidth at the focus is therefore about 6.6^{o}.

The receiver will consist of a cooled photomultiplier with the necessary optical controls at the input and, perhaps, an output amplifier.

The optical controls will include an aperture in the focal plane of the mirror, collimating lenses if necessary, filters and a shutter. All these elements must be adjustable or changeable. The focal-plane stop will be used to define the beamwidth and should therefore provide apertures from 1 to 20 mm in diameter. The basic filter will be, of course, the 20 Å interference filter already mentioned. However, other bandwidths may sometimes be desired, and additional attenuation may be necessary for low-altitude observations, or even daylight observations. Provision must therefore be made to change the filter or use several filters in combination.

The shutter will be necessary for the following reason. The backscatter from the lower atmosphere is so intense that the signals fed to the phototube produce in it a condition of high noise that prevails for sometime after the signal decreases, obscuring the return from greater heights. For operation at maximum sensitivity, then, a shutter must cover the photocathode during the first part of the scan. It must open itself with a precise time lag relative to the laser pulse; for example, $100\,\mu s$ later (this would accept all echoes originated at 15 km or higher). In fact, this time lag should be adjustable at least in a few steps, to allow for observations at different heights. Of course, for low-level observations, neutral filters would be added.

Next comes the photomultiplier and the first task here is to select one. Two characteristics of phototubes have a special bearing on our experiment -- in fact, they appear in our equations. One is the dark current which, when measured at the cathode and expressed in photoelectrons per second, we call N_d in eq. (11). The other is the quantum efficiency, the proportion of the incoming photons that will appear as output pulses. It is included in the factor α, in eqs. (15), (16), etc. In other words, both the signal count N and the background count N_b are proportional to the quantum efficiency, while the tube noise N_d is in itself the dark current.

Being a constant, characteristic of every type of cathode coating, the quantum efficiency provides a first selection criterion, inasmuch as for most surfaces, it is vanishingly small at the ruby laser frequency. Indeed, at $\lambda = 0.7\mu$, the highest efficiency among standard surfaces is found in the S-20 type, and that is only 2.5%. We shall, however, consider a couple of alternatives.

The dark current, on the other hand, is a more complex phenomenon. It arises from many sources, differs from tube to tube and, for a given tube, varies with the overall applied voltage. However, in the normal operating range of any tube, it is caused mainly by thermionic emission from the photo-cathode and is therefore very sensitive to the temperature of the latter (1). In fact, as the temperature is decreased, the dark current decreases sharply, until it eventually reaches a lower limit where it no longer responds to further cooling. The shape of the curve, the temperature at which it levels off and the dark current at this point, depend on the type of cathode. For the S-20 type the knee is at about $-40^{\circ}C$ and the current stays at about 1 electron/cm^2sec. For other surfaces these figures change slightly but, in most cases, it can be estimated that a reduction of a factor of 1000 is brought about by cooling the cathode from room temperature to $-40^{\circ}C$. Perhaps the main difficulty lies in the fact that the temperature of the cathode is not easy to assess and may differ appreciably from that of the cooling fluid. For this reason, it may be found necessary to resort to liquid nitrogen to cool the tube or the cooling gas for the tube, even though the cathode does not have to operate at such a low temperature.

(1) See: Multi Photo-Tube Characteristics: Application to Low Light Level, Ralph W. Engstrom. J. Opt. Soc. Am., Vol. 37, No. 6, 1947, or, more recently: Dark Current in Photomultiplier Tubes. J. Sharpe, Doc. Ref. CP5475, EMI, 1964

The following table lists several photomultipliers made by two American and one British manufacturer. The type of cathode surface and its quantum efficiency are shown. Also shown are the dark count at one indicated applied voltage, and the values used to calculate it. Manufacturers state the anode-dark current of their tubes (which depends on the amplification) rather than the cathode dark current. They also specify the gain, but often both values are given for entirely different operating conditions and cannot be used to calculate the cathode dark current. Values in this table were read off characteristic curves or tables for one operating point, indentified by the applied voltage. The dark count - at room temperature - is the cathode dark current divided by the charge of the electron, 1.6×10^{-19} coulombs.

Manufacturer	Type	Cathode Surface	Quantum Efficiency † %	Applied Voltage volts	Gain	Dark Current Anode Amperes	Dark Current Cathode Amperes	Dark Count count/sec (uncooled)	Price US$
RCA	C70038D	special	5.3	1200	3×10^4	10^{-9}	3.3×10^{-14}	2.1×10^5	
	8644	S-20	2.5	1820	3.5×10^5	2×10^{-9}	5.7×10^{-15}	3.6×10^4	
	7265	S-20	2.5	2400	2×10^7	6×10^{-7}	3×10^{-14}	1.9×10^5	425
EMI	9558B	S-20	2.5	1450	1.4×10^6	10^{-9}	7.1×10^{-16}	4.4×10^3	475
	9529B	S-10	1.0	1250	5.7×10^6	7×10^{-9}	1.2×10^{-15}	7.5×10^3	237
ITT	FW130	S-20	2.5	1800	8×10^5	10^{-8}	1.25×10^{-14}	7.8×10^4	695
	F4003	S-20	2.5	1800	* 5×10^6	* 10^{-10}	2×10^{-17}	125	1375

† At 6943 Å

* It is not unmistakably clear in the technical data that these two values correspond to the same operating conditions

Of these tubes, the first has twice the quantum efficiency of all the others. As for the dark count, the C70038D has the highest. The 9558B - a variant of the tube used by Fiocco and by Wright - has 500 times less dark count, and the F4003 even less. But by adequate cooling, even the dark count of the C70038D can be made small relative to N.

If there is any doubt as to what tube to choose, this can be tested with the help of eqs. (11), (15), and (16).

Let us apply them to the case of our 18" astronomical mirror, where dark count would be of greater importance. We have, then: $A_r = 0.164 m^2$; $\Omega = 7.85 \times 10^{-7}$ sr; $B = 20$ Å; $\tau = 10^{-5}$ s. As before, $\alpha = \alpha_F \cdot \alpha_{pht}$, where we take the filter transmission $\alpha_F = 0.5$ and α_{pht} is now the quantum efficiency at the laser frequency.

For the C70038D, the tube with the highest quantum efficiency and the highest dark count, we find:

$\alpha_{pht} = 0.053$; $N = 1890$ ct/s; $N_b = 143$ ct/s.

We assume that by cooling the dark count is reduced 1000 times, making: $N_d = 200$ counts/sec. Then, for $P = 10$:

$$u = 100 \frac{1890 + 143 + 200}{10^{-5} \times 3.56 \times 10^{6}} = 6280$$

The other extreme case is that of the F4003. Assuming that our values are correct, the dark count can be reduced by cooling to less than 1 count per second (which is made possible by the fact that this type of tube uses only a very small part of its cathode at any given time). We may as well assume the ideal case: $N_d = 0$. But now $\alpha_{pht} = 0.025$. Then:

Then:

$$N = 900 \text{ ct/s}; \quad N_b = 67 \text{ ct/s}; \text{ and } u = 100 \frac{900 + 67}{10^{-5} \times 8.1 \times 10^{5}} \approx 12000$$

The conclusion is clear: the higher quantum efficiency prevails. Our choice is the RCA C70038D.

System Comparison

The purpose of this section is to judge the relative merits of four instrumental set-ups, by calculating the number of pulses it would take, with each of them, to detect our hypothetical dust layer with a statistical significance of ten times the total fluctuation.

The four systems compared are: the one used by Fiocco and Smullin, the one used by Wright and the two we are proposing here: with the 18" astronomical mirror and with the less accurate 40" mirror.

Fiocco's instrument (1) was based on a ruby laser capable of 0.5 Joule pulses. He collected his backscattered light on a mirror of 0.08 m^2 area and integrated it over 66.666... μs intervals. It was fed through a 20 Å filter to a cooled EMI 9558 A photomultiplier. Though the temperature of the cathode is not stated in his report, it can be inferred from other texts (2) that it was cooled beyond -40°C, and EMI has published literature showing that the dark current of the 9558 levels off at that temperature, with a count of about 15 photoelectrons per second. (3) Its quantum efficiency, at the ruby laser frequency, is 2.5%.

If we further assume the peak transmission of the filter to be 50%, we have all the necessary values to apply eq. (9) and finally obtain, with our assumed cross-section: N = 44 counts/s.

We cannot calculate the background because the field of view of the receiver is not given in the report. But we can estimate it from the results, where it

(1) as described in Nature, 199, p. 1275, 1963.
(2) Proc. IRE, 50, p. 1703. 1962.
(3) EMI Document No. CP 5475: Dark current in photomultiplier tubes by J. Sharpe. 1964.

appears to have been shown as noise, together with the dark-current. By averaging the data taken on July 28th, 30th, and 31st, 1963, we find: $N_b + N_d = 2330$ counts/s. If we now solve eq. (11) for a ratio $P = 10$ and an integration time of $10\,\mu$s, we get: $u = 12.2 \times 10^6$ pulses.

Actually, Fiocco's conclusions are based on much shorter series of readings, because: 1) evidently, the cross-section of the observed layer was about 100 times greater than we estimated here, as can be found by using his results ($N \approx 3500$ ct/s) in eq. (7), to calculate $n\sigma$. 2) He accepted counts better than 3 times the total fluctuation. 3) His counts were integrated over 66.6... μs intervals, i.e. six times longer than ours.

If we turn to Wright's experiment, we find improvements in almost all parameters: the laser energy per pulse was 5 Joules and the area of the collecting mirror was $0.2\,m^2$; these two changes should increase the signal count about 25 times. The phototube was again the 9558A and appears to have been carefully cooled; yet Wright admits about 50 counts per second on account of dark current. Let us again take 2.5% for the quantum efficiency. Wright gives 0.6 for the peak transmission of his filter, which again had a 20 Å band. With these values, eq. (9) yields: $N = 1330$ counts per second.

Since the aperture of the receiver is given in the report, we can calculate the solid angle, which turns out to be $\Omega = 1.26 \times 10^{-7}$ sr, whence the background: $N_b = 16$ ct/s. Here, the great improvement over Fiocco's results, probably stems from a much smaller aperture. If we use eq. (16) to estimate Ω in Fiocco's experiment, on the basis of the known background, we find: $\Omega = 5.5 \times 10^{-5}$, i.e., about 400 times greater than Wright's. That eq. (16), on the other hand, is indeed a fair representation of the experiment, is confirmed by the fact that Wright's measured background is stated to be "about 10 ct/s".

Wright used an integrating time of only $10 \mu s$. While this naturally reduced all his counts, it had the great advantage of improving the range-resolution, at the cost of some sacrifice in sensitivity, that he could well afford.

We can now calculate u by means of eq. (11) and, for P = 10, we find: u = 7900.

Our proposed experiment, with the 18 inch astronomical mirror, was already appraised when selecting the phototube. For comparison we do the same with the larger mirror suggested as an alternative. The results for all four cases and the relevant parameters are presented in the following table, where all systems are compared on the basis of a target cross-section of $3 \times 10^{-11} \frac{m^2}{m^3}$, at a height of 100 km, observed with an integrating interval of 10^{-5} sec, through a 20 Å filter. Then u is the number of pulses necessary to obtain a ratio of 10 between signal count and total fluctuation.

Parameter	Unit	Fiocco et al.	Wright et al.	U. of Miami astronomical mirror	U. of Miami large mirror
Laser Energy E_o	Joules	0.5	5	5	5
Mirror Area A_r	m^2	0.08	0.2	0.164	0.81
Solid Angle Ω	sr	*5.5×10^{-5}	1.26×10^{-7}	7.85×10^{-7}	6×10^{-6}
Filter Transm. α_F	%	50	60	50	50
Phototube	--	9558A	9558A	C70038D	C70038D
Quantum Effic. α_{pht}	%	2.5	2.5	5.3	5.3
Signal N	$\frac{\text{counts}}{\text{sec}}$	44	1330	1890	9450
Background N_b	$\frac{\text{counts}}{\text{sec}}$	2310	16	143	5400
Tube Noise N_d	$\frac{\text{counts}}{\text{sec}}$	20	50	200	200
Nr. of pulses u	--	12.2×10^6	7900	6280	1680

* Not given in the report, but calculated from the recorded background.

Atmospheric Scattering

Molecular scattering by atmospheric gases is important in our experiment for two reasons: first because it may provide a remote measurement of atmospheric density, and secondly because it will tend to obscure the presence of other scatterers, such as dust particles, if the latter have smaller backscattering cross-sections than atmospheric gas at the same level, on a unit-volume basis. Also, molecular scattering is of course partly responsible for the attenuation of our laser ray along its path.

We shall therefore calculate the amount of atmospheric scattering that we must expect.

The backscattering cross-section of a spherical particle of radius small compared to the wavelength, is given by the familiar theory developed by Rayleigh. We define it as: "The area intercepting on the incident wavefront that amount of power which, when scattered isotropically, produces at the radiation source the observed echo".[1] This is also called the "radar cross-section". In our derivations we designated it σ. For the Rayleigh case we have:

$$\sigma = \frac{64\pi^5}{\lambda^4} \left| \frac{m^2 - 1}{m^2 + 2} \right|^2 a^6 \tag{24}$$

where λ and a are the wavelength and the radius of the particle in any consistent system of units, m being the complex refractive index. The bars indicate that the magnitude of the complex ratio is to be considered; however, for a gas the absorption is negligible, m becomes the usual real refractive index, and what we have becomes the square of a real quantity.

Eq. (24) is only valid for spheres. For them:

$$a^6 = \frac{9}{16} \frac{v^2}{\pi^2}$$

where v is the volume of every particle. Substituting in eq. (24) we get:

$$\sigma = \frac{36\pi^3}{\lambda^4} \left(\frac{m^2 - 1}{m^2 + 2} \right)^2 v^2 \tag{25}$$

(1) D. E. Kerr, Propagation of short radio waves, Radiation Lab. series, vol. XIII, McGraw-Hill, 1951, p. 33. See also Chap. 6.

For a gas, however, m is sufficiently close to unity to justify the following approximation: [1]

$$\left(\frac{m^2 - 1}{m^2 + 2}\right)^2 = \frac{(m + 1)^2 (m - 1)^2}{(m^2 + 2)^2} \approx \frac{4}{9} (m - 1)^2$$

which, when substituted in eq. (25), leads to:

$$\sigma = \frac{16\pi^3}{\lambda^4} (m - 1)^2 v^2 \qquad (26)$$

Here, then, m and v correspond to individual molecules and we would have to know them in order to use eq. (26) directly. However, we know that the product $(m - 1)v$ is a constant for every gas and, in fact, the macroscopic index of refraction of a gas, at a given density, is determined by this constant. If we call it m_{gas}, we have:

$$m_{gas} - 1 = (m - 1)vn = kn \qquad (27)$$

where n is the number of molecules per unit volume.

Now the radar cross-section of the atmosphere at a given height, per unit volume, will be n times the cross-section of every molecule, if n is the number density at that height. Then, combining egs. (26) and (27), we can write:

$$n\sigma = \frac{16\pi^3}{\lambda^4} k^2 n \qquad (28)$$

(1) J. C. Johnson, Physical Meteorology, Chap. II

This expression is now calculable because k can be found from eq. (27) and a measurement of the macroscopic refractive index and the density. In fact, it is known that at sea level:

$$m_{air} - 1 = 293 \times 10^{-6} \text{ when } n = 2.66 \times 10^{25} \frac{\text{molecules}}{m^3}$$

Then k, which has the dimensions of a volume, is equal to 1.1×10^{-29} m^3. This, however, is only true as long as the composition of the atmosphere does not change appreciably. Therefore, it is no longer valid where molecular dissociation prevails; there, eq. (28) leads us to expect a decrease in cross-section, k being proportional to the volume of the scatterers. The amount of dissociation is indicated by the change in mean molecular weight of the atmosphere. Therefore, in the following table this magnitude is also shown, in order to provide an indication of the validity of the other figures.

As long as the value given for k holds, eq. (28) can be written, for $\lambda = 7 \times 10^{-7}$ m:

$$n\sigma = 2.5 \times 10^{-31} n \quad \frac{m^2}{m^3} \tag{29}$$

The following table shows the backscattering cross-section per unit volume of atmosphere, $n\sigma$, as a function of height. It also shows the signal count expected in our proposed receivers, with the astronomical mirror and with the larger mirror. These counts were calculated with eq. (7), since the height varies from one value to the next. However, $T^2(z,\lambda)$ was taken as constant and equal to 0.7 for all of them, because by far the heaviest attenuation takes place in the first 20 km, i.e. before the first value. Finally, the number of pulses necessary to yield a measurement with a signal-to-fluctuation ratio of 10, is computed for both systems.

Altitude km	Mean Molecular Weight	Number Density per m^3	Radar Cross-Section, m^{-1}	Astronomical Mirror		Large Mirror	
				N, $\frac{\text{counts}}{\text{sec}}$	u, pulses	N, $\frac{\text{counts}}{\text{sec}}$	u, pulses
0	28.966	2.55×10^{25}	6.38×10^{-6}				
20	28.97	1.85×10^{24}	4.63×10^{-7}	7.43×10^{8}	1	3.67×10^{9}	1
40	28.97	8.46×10^{22}	2.12×10^{-8}	8.49×10^{6}	2	4.19×10^{7}	1
60	28.97	6.33×10^{21}	1.59×10^{-9}	2.82×10^{5}	36	1.39×10^{6}	8
80	28.97	4.03×10^{20}	1.01×10^{-10}	1.01×10^{4}	1023	4.99×10^{4}	224
100	28.85	9.98×10^{18}	2.50×10^{-12}	160	1.98×10^{5}	790	1.03×10^{5}
120	28.60	5.15×10^{17}	1.29×10^{-13}	5.73	9.7×10^{7}	28.3	7.03×10^{7}
140	28.25	6.55×10^{16}	1.64×10^{-14}	0.53		2.64	

Values of Mean Molecular Weight and Number Density were taken from Cospar International Reference Atmosphere, 1961, prepared by H. Kallmann-Bijl, R.L.F. Boyd, H. Lagow, S. M. Poloskov and W. Priester.

Inspection of this table reveals several facts that have a bearing on the present and future possibilities of this experiment. Let us point at a few:

1) Comparison of this table with the previous one shows that the expected dust layer at 100 km will produce a signal count about 10 times greater than would molecular scattering and will therefore not be overshadowed by the atmosphere. And yet, Fiocco's results seem to indicate that we are here underestimating the cross-section of the dust layer.

2) As the height increases, the signal count drops very rapidly, on account of greater range and smaller density. As a consequence, the number of necessary pulses grows also very rapidly. At about 100 km, it is so great with the systems presently proposed, that the measurement of atmospheric scattering ceases to be a practical possibility: it would take at least about 30 hours, which is too long a time in view of the diurnal changes that take place at that height. This effect may be even worse than reflected in the table, if molecular dissociation of oxygen further reduces the backscatter above 90 km.

3) As the received signal decreases, it becomes gradually immaterial what size of mirror is used to collect it. From 60 to 80 km, the larger mirror requires about 5 times less pulses than the small one for the same statistical validity in the measurement. Above these altitudes, the ratio decreases; it is less than 1.4 at 120 km.